

# Landau Gauge QCD: Functional Methods versus Lattice Simulations<sup>†</sup>

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## Abstract

The infrared behaviour of QCD Green's functions in Landau gauge has been focus of intense study. Different non-perturbative approaches lead to a prediction in line with the conditions for confinement in local quantum field theory as spelled out in the Kugo-Ojima criterion. Detailed comparisons with lattice studies have revealed small but significant differences, however. But aren't we comparing apples with oranges when contrasting lattice Landau gauge simulations with these continuum results? The answer is yes, and we need to change that. We therefore propose a reformulation of Landau gauge on the lattice which will allow us to perform gauge-fixed Monte-Carlo simulations matching the continuum methods of local field theory which will thereby be elevated to a truly non-perturbative level at the same time.

## Introduction

The Green's functions of QCD are the fundamental building blocks of hadron phenomenology [1]. Their infrared behaviour is also known to contain essential information about the realisation of confinement in the covariant formulation of QCD, in terms of local quark and gluon field systems. The Landau gauge Dyson-Schwinger equation (DSE) studies of Refs. [2, 3] established that the gluon propagator alone does not provide long-range interactions of a strength sufficient to confine quarks. This dismissed a widespread conjecture from the 1970's going back to the work of Marciano, Pagels, Mandelstam and others. The idea was revisited that the infrared dominant correlations are instead mediated by the Faddeev-Popov ghosts of this formulation, whose propagator was found to be infrared enhanced. This infrared behaviour is now completely understood in terms of confinement in QCD [1, 4, 5], it is a consequence of the celebrated Kugo-Ojima (KO) confinement criterion.

This criterion is based on the realization of the unfixed global gauge symmetries of the covariant continuum formulation. In short, two conditions are required by the KO criterion to distinguish confinement from Coulomb and Higgs phases: (a) The massless single particle singularity in the transverse gluon correlations of perturbation theory must be screened non-perturbatively to avoid long-range fields and charged superselection sectors as in QED. (b) The global gauge charges must remain well-defined and unbroken to avoid the Higgs mechanism. In Landau gauge, in which the (Euclidean) gluon and ghost propagators,

$$D_{\mu\nu}^{ab}(p) = \delta^{ab} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}, \quad \text{and} \quad D_G^{ab}(p) = -\delta^{ab} \frac{G(p^2)}{p^2}, \quad (1)$$

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are parametrised by the two invariant functions  $Z$  and  $G$ , respectively, this criterion requires

$$(a): \lim_{p^2 \rightarrow 0} Z(p^2)/p^2 < \infty ; \quad (b): \lim_{p^2 \rightarrow 0} G^{-1}(p^2) = 0 . \quad (2)$$

The translation of (b) into the infrared enhancement of the ghost propagator (2b) thereby rests on the ghost/anti-ghost symmetry of the Landau gauge or the symmetric Curci-Ferrari gauges. In particular, this equivalence does not hold in linear covariant gauges with non-zero gauge parameter such as the Feynman gauge.

As pointed out in [5], the infrared enhancement of the ghost propagator (2b) represents an additional boundary condition on DSE solutions which then lead to the prediction of a conformal infrared behaviour for the gluonic correlations in Landau gauge QCD consistent with the conditions for confinement in local quantum field theory. In fact, this behaviour is directly tied to the validity and applicability of the framework of local quantum field theory for non-Abelian gauge theories beyond perturbation theory. The subsequent verification of this infrared behaviour with a variety of different functional methods in the continuum meant a remarkable success. These methods which all lead to the same prediction include studies of their Dyson-Schwinger Equations (DSEs) [5], Stochastic Quantisation [6], and of the Functional Renormalisation Group Equations (FRGEs) [7]. This prediction amounts to infrared asymptotic forms

$$Z(p^2) \sim (p^2/\Lambda_{\text{QCD}}^2)^{2\kappa_Z} , \quad \text{and} \quad G(p^2) \sim (p^2/\Lambda_{\text{QCD}}^2)^{-\kappa_G} , \quad (3)$$

for  $p^2 \rightarrow 0$ , which are both determined by a unique critical infrared exponent

$$\kappa_Z = \kappa_G \equiv \kappa , \quad (4)$$

with  $0.5 < \kappa < 1$ . Under a mild regularity assumption on the ghost-gluon vertex [5], the value of this exponent is furthermore obtained as [5, 6]

$$\kappa = (93 - \sqrt{1201})/98 \approx 0.595 . \quad (5)$$

The conformal nature of this infrared behaviour in the pure Yang-Mills sector of Landau gauge QCD is evident in the generalisation to arbitrary gluonic correlations [8]: a uniform infrared limit of one-particle irreducible vertex functions  $\Gamma^{m,n}$  with  $m$  external gluon legs and  $n$  pairs of ghost/anti-ghost legs of the form

$$\Gamma^{m,n} \sim (p^2/\Lambda_{\text{QCD}}^2)^{(n-m)\kappa} , \quad (6)$$

when all  $p_i^2 \propto p^2 \rightarrow 0$ ,  $i = 1, \dots, 2n + m$ . In particular, the ghost-gluon vertex is then infrared finite (with  $n = m = 1$ ) as it must [9], and the non-perturbative running coupling introduced in [2, 3] via the definition

$$\alpha_S(p^2) = \frac{g^2}{4\pi} Z(p^2) G^2(p^2) \quad (7)$$

approaches an infrared fixed-point,  $\alpha_S \rightarrow \alpha_c$  for  $p^2 \rightarrow 0$ . If the ghost-gluon vertex is regular at  $p^2 = 0$ , its value is maximised and given by [5]

$$\alpha_c = \frac{8\pi}{N_c} \frac{\Gamma^2(\kappa - 1)\Gamma(4 - 2\kappa)}{\Gamma^2(-\kappa)\Gamma(2\kappa - 1)} \approx \frac{9}{N_c} \times 0.99 . \quad (8)$$

Comparing the infrared scaling behaviour of DSE and FRGE solutions of the form of Eqs. (3), it has in fact been shown that in presence of a single scale, the QCD scale  $\Lambda_{\text{QCD}}$ , the solution with the infrared behaviour (4) and (6), with a positive exponent  $\kappa$ , is unique [10]. Because of its uniqueness, it is nowadays being called the *scaling solution*.

This uniqueness proof does not rule out, however, the possibility of a solution with an infrared finite gluon propagator, as arising from a transverse gluon mass  $M$ , which then leads to an essentially free ghost propagator, with the free massless-particle singularity at  $p^2 = 0$ , *i.e.*,

$$Z(p^2) \sim p^2/M^2, \text{ and } G(p^2) \sim \text{const.} \quad (9)$$

for  $p^2 \rightarrow 0$ . The constant contribution to the zero-momentum gluon propagator,  $D(0) = 3/(4M^2)$ , thereby necessarily leads to an infrared constant ghost renormalisation function  $G$ . This solution corresponds to  $\kappa_Z = 1/2$  and  $\kappa_G = 0$ . It does not satisfy the scaling relations (4) or (6). This is because in this case the transverse gluons decouple for momenta  $p^2 \ll M^2$ , below the independent second scale given by their mass  $M$ . It is thus not within the class of scaling solutions considered above, and it is termed the *decoupling solution* in contradistinction [11]. The interpretation of the renormalisation group invariant (7) as a running coupling does not make sense in the infrared in this case, in which there is no infrared fixed-point and no conformal infrared behaviour.

Without infrared enhancement of the ghosts in Landau gauge, the global gauge charges of covariant gauge theory are spontaneously broken. Within the language of local quantum field theory the decoupling solution can thus only be realised if and only if it comes along with a Higgs mechanism and massive physical gauge bosons. The Schwinger mechanism can in fact be described in this way, and it can furthermore be shown that a non-vanishing gauge-boson mass, by whatever mechanism it is generated, necessarily implies the spontaneous breakdown of global symmetries [12].

## Landau Gauge QCD in the Continuum and on the Lattice

Early lattice studies of the gluon and ghost propagators supported their predicted infrared behaviour qualitatively well. Because of the inevitable finite-volume effects, however, these results could have been consistent with both, the scaling solution as well as the decoupling solution. Recently, the finite-volume effects have been analysed carefully in the Dyson-Schwinger equations to demonstrate how the scaling solution is approached in the infinite volume limit there [13]. Comparing these finite volume DSE results with latest  $SU(2)$  lattice data on impressively large lattices [14, 15], corresponding to physical lengths of up to 20 fm in each direction, finite-volume effects appear to be ruled out as the dominant cause of the observed discrepancies with the scaling solution. The lattice results are much more consistent with the decoupling solution which poses the obvious question whether there is something wrong with our general understanding of covariant gauge theory or whether we are perhaps comparing apples with oranges when applying inferences drawn from the infrared behaviour of the lattice Landau gauge correlations on local quantum field theory?

The latter language is based on a cohomology construction of a physical Hilbert space over the indefinite metric spaces of covariant gauge theory from the representations of the Becchi-Rouet-Stora-Tyutin (BRST) symmetry. But do we have a non-perturbative definition of a BRST charge? The obstacle is the existence of the so-called Gribov copies

which satisfy the same gauge-fixing condition, *i.e.*, the Lorenz condition in Landau gauge, but are related by gauge transformations, and are thus physically equivalent. In fact, in the direct translation of BRST symmetry on the lattice, there is a perfect cancellation among these gauge copies which gives rise to the famous Neuberger 0/0 problem. It asserts that the expectation value of any gauge invariant (and thus physical) observable in a lattice BRST formulation will always be of the indefinite form 0/0 [16] and therefore prevented such formulations for more than 20 years now.

In present lattice implementations of the Landau gauge this problem is avoided because the numerical procedures are based on minimisations of a gauge fixing potential w.r.t. gauge transformations. To find absolute minima is not feasible on large lattices as this is a non-polynomially hard computational problem. One therefore settles for local minima which in one way or another, depending on the algorithm, samples gauge copies of the first Gribov region among which there is no cancellation. For the same reason, however, this is not a BRST formulation. The emergence of the decoupling solution can thus not be used to dismiss the KO criterion of covariant gauge theory in the continuum.

## Strong Coupling Limit of Lattice Landau Gauge

From the finite-volume DSE solutions of [13] it follows that a wide separation of scales is necessary before one can even hope to observe the onset of an at least approximate conformal behaviour of the correlation functions in a finite volume of length  $L$ . What is needed is a reasonably large number of modes with momenta  $p$  sufficiently far below the QCD scale  $\Lambda_{\text{QCD}}$  whose corresponding wavelengths are all at the same time much shorter than the finite size  $L$ ,

$$\pi/L \ll p \ll \Lambda_{\text{QCD}} . \quad (10)$$

It was estimated that this requires sizes  $L$  of about 15 fm, especially for a power law of the ghost propagator of the form in (3) to emerge in a momentum range with (10). A reliable quantitative determination of the exponents and a verification of their scaling relation (4) on the other hand might even require up to  $L = 40$  fm [13].

As an alternative to the brute-force method of using ever larger lattice sizes for the simulations might therefore be to ask what one observes when the formal limit  $\Lambda_{\text{QCD}} \rightarrow \infty$  is implemented by hand. This should then allow to assess whether the predicted conformal behaviour can be seen for the larger lattice momenta  $p$ , after the upper bound in (10) has been removed, in a range where the dynamics due to the gauge action would otherwise dominate and cover it up completely. Therefore, the ghost and gluon propagators of pure  $SU(2)$  lattice Landau gauge were studied in the strong coupling limit  $\beta \rightarrow 0$  in [17, 18].

In this limit, the gluon and ghost dressing functions tend towards the decoupling solution at small momenta and towards the scaling solution at large momenta (in units of the lattice spacing  $a$ ) as seen in Figure 1. The transition from decoupling to scaling occurs at around  $a^2 p^2 \approx 1$ , independent of the size of the lattice. The observed deviation from scaling at  $a^2 p^2 < 1$  is thus not a finite-size effect. The high momentum branch can be used to attempt fits of  $\kappa_Z$  and  $\kappa_G$  in (3) and the data is consistent with the scaling relation (4). With some dependence on the model used to fit the data, good global fits are generally obtained for  $\kappa = 0.57(3)$ , with very little dependence on the lattice size.

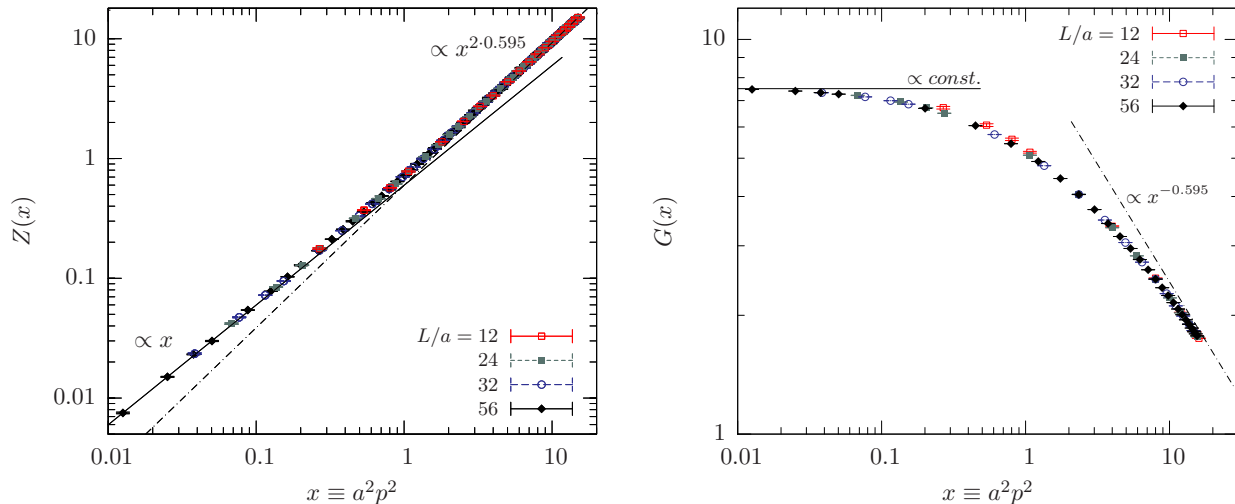


Figure 1: The gluon (left) and ghost (right) dressing functions at  $\beta = 0$  compared to the decoupling solution (solid) and the scaling solution (dashed) with  $\kappa$  from (5) (not fitted).

For the scaling solution one would expect the running coupling defined by (7) to approach its constant fixed-point value in the strong-coupling limit, and this is indeed being observed for the scaling branch [17]: The numerical data for the product (7) levels at  $\alpha_c \approx 4$  for large  $a^2 p^2$ . As expected for an exponent  $\kappa$  slightly smaller than the value in (5), see [5], this is just below the upper bound given by (8),  $\alpha_c \approx 4.45$  for  $SU(2)$ .

When comparing various definitions of gauge fields on the lattice, all equivalent in the continuum limit, one furthermore observes that neither the estimate of the critical exponent  $\kappa$  nor the corresponding value of  $\alpha_c$  are sensitive to the definition used [17]. This is in contrast to the decoupling branch for  $a^2 p^2 < 1$ , which is very sensitive to that definition. Different definitions, at order  $a^2$  and beyond, lead to different Jacobian factors. This is well known from lattice perturbation theory where, however, the lattice Slavnov-Taylor identities guarantee that the gluon remains massless at every order by cancellation of all quadratically divergent contributions to its self-energy. The strong-coupling limit, where the effective mass in (9) behaves as  $M^2 \propto 1/a^2$ , therefore shows that such a contribution survives non-perturbatively in minimal lattice Landau gauge. This contribution furthermore depends on the measure for gauge fields whose definition from minimal lattice Landau is therefore ambiguous. One might still hope that this ambiguity will go away at non-zero  $\beta$ , in the scaling limit. While this is true at large momenta, it is not the case in the infrared, at least not for commonly used values of the lattice coupling such as  $\beta = 2.5$  or  $\beta = 2.3$  in  $SU(2)$ , as demonstrated in [17].

## Lattice BRST and the Neuberger 0/0 Problem

It would obviously be desirable to have a BRST symmetry on the lattice which could then provide lattice Slavnov-Taylor identities beyond perturbation theory. In principle, this could be achieved by inserting the partition function of a topological model with BRST exact action into the gauge invariant lattice measure. Because of its topological nature, this

gauge-fixing partition function  $Z_{\text{GF}}$  will be independent of gauge orbit and gauge parameter. The problem is that in the standard formulation this partition function calculates the Euler characteristic  $\chi$  of the lattice gauge group which vanishes [19],

$$Z_{\text{GF}} = \chi(SU(N)^{\#\text{sites}}) = \chi(SU(N))^{\#\text{sites}} = 0^{\#\text{sites}}. \quad (11)$$

Neuberger's 0/0 problem of lattice BRST arises because we have then inserted zero instead of unity (according to the Faddeev-Popov prescription) into the measure of lattice gauge theory. On a finite lattice, such a topological model is equivalent to a problem of supersymmetric quantum mechanics with Witten index  $\mathcal{W} = Z_{\text{GF}}$ . Unlike the case of primary interest in supersymmetric quantum mechanics, here we need a model with non-vanishing Witten index to avoid the Neuberger 0/0 problem. Then however, just as the supersymmetry of the corresponding quantum mechanical model, such a lattice BRST cannot break.

In Landau gauge, with gauge parameter  $\xi = 0$ , the Neuberger zero,  $Z_{\text{GF}} = 0$ , arises from the perfect cancellation of Gribov copies via the Poincaré-Hopf theorem. The gauge-fixing potential  $V_U[g]$  for a generic link configuration  $\{U\}$  thereby plays the role of a Morse potential for gauge transformations  $g$  and the Gribov copies are its critical points (the global gauge transformations need to remain unfixed so that there are strictly speaking only  $(\#\text{sites}-1)$  factors of  $\chi(SU(N)) = 0$  in (11)). The Morse inequalities then immediately imply that there are at least  $2^{(N-1)(\#\text{sites}-1)}$  such copies in  $SU(N)$  on the lattice, or  $2^{\#\text{sites}-1}$  in compact  $U(1)$ , and equally many with either sign of the Faddeev-Popov determinant (*i.e.*, that of the Hessian of  $V_U[g]$ ).

The topological origin of the zero originally observed by Neuberger in a certain parameter limit due to uncompensated Grassmann ghost integrations in standard Faddeev-Popov theory [16] becomes particularly evident in the ghost/anti-ghost symmetric Curci-Ferrari gauge with its quartic ghost self-interactions [20]. Due to its Riemannian geometry with symmetric connection and curvature tensor  $R_{ijkl} = \frac{1}{4}f^a_{ij}f^a_{kl}$  for  $SU(N)$ , in this gauge the same parameter limit leads to computing the zero in (11) from a product of independent Gauss-Bonnet integral expressions,

$$\chi(SU(N)) = \frac{1}{(2\pi)^{(N^2-1)/2}} \int_{SU(N)} dg \int d\bar{c} dc \exp \left\{ \frac{1}{4} R_{abcd} \bar{c}^a c^b c^c c^d \right\} = 0, \quad (12)$$

for each site of the lattice. This corresponds to the Gauss-Bonnet limit of the equivalent supersymmetric quantum mechanics model in which only constant paths contribute [21].

The indeterminate form of physical observables as a consequence of (12) is regulated by a Curci-Ferrari mass term. While such a mass  $m$  decontracts the double BRST/anti-BRST algebra, which is well-known to result in a loss of unitarity, observables can then be meaningfully defined in the limit  $m \rightarrow 0$  via l'Hospital's rule [20].

## Lattice Landau Gauge from Stereographic Projection

The 0/0 problem due to the vanishing Euler characteristic of  $SU(N)$  is avoided when fixing the gauge only up to the maximal Abelian subgroup  $U(1)^{N-1}$  because the Euler characteristic of the coset manifold is non-zero. The corresponding lattice BRST has been explicitly constructed for  $SU(2)$  [19], where the coset manifold is the 2-sphere and  $\chi(SU(2)/U(1)) = \chi(S^2) = 2$ . This indicates that the Neuberger problem might be solved

when that of compact  $U(1)$  is, where the same cancellation of lattice Gribov copies arises because  $\chi(S^1) = 0$ . A surprisingly simple solution to this problem is possible, however, by stereographically projecting the circle  $S^1 \rightarrow \mathbb{R}$  which can be achieved by a simple modification of the minimising potential [22]. The resulting potential is convex to the above and leads to a positive definite Faddeev-Popov operator for compact  $U(1)$  where there is thus no cancellation of Gribov copies, but  $Z_{\text{GF}}^{U(1)} = N_{\text{GC}}$ , for  $N_{\text{GC}}$  Gribov copies.

As compared to the standard lattice Landau gauge for compact  $U(1)$  their number is furthermore exponentially reduced. This is easily verified explicitly in low dimensional models. While  $N_{\text{GC}}$  grows exponentially with the number of sites in the standard case as expected, the stereographically projected version has only  $N_{\text{GC}} = N_x$  copies on a periodic chain of length  $N_x$  and  $\ln N_{\text{GC}} \sim N_t \ln N_x$  on a  $2D$  lattice of size  $N_t N_x$  in Coulomb gauge, for example, and in both cases their number is verified to be independent of the gauge orbit.

The general proof of  $Z_{\text{GF}}^{U(1)} = N_{\text{GC}}$  with stereographic projection which avoids the Neuberger zero in compact  $U(1)$  [22] follows from a simple example of a Nicolai map [21].

Applying the same techniques to the maximal Abelian subgroup  $U(1)^{N-1}$ , the generalisation to  $SU(N)$  lattice gauge theories is possible when the odd-dimensional spheres  $S^{2n+1}$ ,  $n = 1, \dots, N-1$ , of its parameter space are stereographically projected to  $\mathbb{R} \times \mathbb{R}P(2n)$ . In absence of the cancellation of the lattice artifact Gribov copies along the  $U(1)$  circles, the remaining cancellations between copies of either sign in  $SU(N)$ , which will persist in the continuum limit, are then necessarily incomplete, however, because  $\chi(\mathbb{R}P(2n)) = 1$ .

For  $SU(2)$  this program is straightforward. One replaces the standard gauge-fixing potential  $V_U[g]$  of lattice Landau gauge by  $\tilde{V}_U[g]$ , via gauge-transformed links  $U_{x\mu}^g$ , where

$$V_U[g] = 4 \sum_{x,\mu} \left( 1 - \frac{1}{2} \text{Tr } U_{x\mu}^g \right) \quad \text{and} \quad \tilde{V}_U[g] = -8 \sum_{x,\mu} \ln \left( \frac{1}{2} + \frac{1}{4} \text{Tr } U_{x,\mu}^g \right). \quad (13)$$

The standard and stereographically projected gauge fields on the lattice are defined as

$$A_{x\mu} = \frac{1}{2ia} (U_{x\mu} - U_{x\mu}^\dagger) \quad \text{and} \quad \tilde{A}_{x\mu} = \frac{1}{2ia} (\tilde{U}_{x\mu} - \tilde{U}_{x\mu}^\dagger), \quad \text{with} \quad \tilde{U}_{x\mu} \equiv \frac{2U_{x\mu}}{1 + \frac{1}{2} \text{Tr } U_{x\mu}}.$$

The gauge-fixing conditions  $F = 0$  and  $\tilde{F} = 0$  are their respective lattice divergences, in the language of lattice cohomology,  $F = \delta A$  and  $\tilde{F} = \delta \tilde{A}$ . A particular advantage of the non-compact  $\tilde{A}$  is that they allow to resolve the modified lattice Landau gauge condition  $\tilde{F} = 0$  by Hodge decomposition. This provides a framework for gauge-fixed Monte-Carlo simulations which is currently being developed for the particularly simple case of  $SU(2)$  in 2 dimensions. In the low-dimensional models mentioned above it can furthermore be verified explicitly that the corresponding topological gauge-fixing partition function is indeed given by

$$Z_{\text{GF}}^{SU(2)} = Z_{\text{GF}}^{U(1)} \neq 0, \quad (14)$$

as expected from  $\chi(\mathbb{R}P(2)) = 1$ . The proof of this will be given elsewhere.

## Conclusions and Outlook

Comparisons of the infrared behaviour of QCD Green's functions as obtained from lattice Landau gauge implementations based on minimisations of a gauge-fixing potential and

from continuum studies based on BRST symmetry have to be taken with a grain of salt. Evidence of the asymptotic conformal behaviour predicted by the latter is seen in the strong coupling limit of lattice Landau gauge where such a behaviour can be observed at large lattice momenta  $a^2 p^2 \gg 1$ . There the strong coupling data is consistent with the predicted critical exponent and coupling from the functional approaches. The deviations from scaling at  $a^2 p^2 < 1$  are not finite-volume effects, but discretisation dependent and hint at a breakdown of BRST symmetry arguments beyond perturbation theory in this approach. Non-perturbative lattice BRST has been plagued by the Neuberger 0/0 problem, but its improved topological understanding provides ways to overcome this problem. The most promising one at this point rests on stereographic projection to define gauge fields on the lattice together with a modified lattice Landau gauge. This new definition has the appealing feature that it will allow gauge-fixed Monte-Carlo simulations in close analogy to the continuum BRST methods which it will thereby elevate to a non-perturbative level.

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